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Journal of Approximation Theory 123 (2003) 1–12

JOURNAL OF
**Approximation
Theory**

<http://www.elsevier.com/locate/jat>

In Memoriam
William Allan Light
(1950–2002)

Prepared by E.W. Cheney and J. Levesley

William Allan Light, known to his friends as “Will,” died on Sunday, December 8, 2002. He was at his home, Gartree House, Illston-on-the-Hill, near Leicester in England, when he was felled by a heart attack. He was 52 years old.

Will is survived by his wife, the former Anita Edwards, by his mother, Mrs. Mary Light of Lancaster, and by a sister, Helen Johnson.

At the time of death, Will was an active member of the Mathematics and Computer Science Department at Leicester University and was a Pro-Vice Chancellor of that University.

Will was born on April 19, 1950, in Chester, England. He attended The University of Sussex and received there, in 1971, the degree of Bachelor of Science with honors. In the next year, he received a Certificate of Education from the University of North Wales and then went to Lancaster University, where he earned a Master of Arts degree (awarded with distinction) in the Department of Mathematics. Remaining at Lancaster, he then embarked on doctoral research under the supervision of Professor Charles Clenshaw. This work culminated in the awarding of the Doctorate of Philosophy in Mathematics in 1976. His dissertation concerned projections in numerical analysis, a theme that he continued to pursue after the doctorate.

Will served first as Lecturer and then as Senior Lecturer in Mathematics at the University of Lancaster. He was noted for the clarity of his lectures and for an ability to keep the attention of his audience. In 1988, Will and his colleague John Gilbert took over the organization of the Science and Engineering Research Council’s Summer Schools in Numerical Analysis at Lancaster. These had been initiated in 1981 by Will’s then colleague, Peter Turner. Many young mathematicians owe their successful careers to the stimulation of these summer schools, in which attendees were brought into contact with pre-eminent experts through a program of lectures on special topics.

In 1991, Will was appointed to a new Chair of Applied Mathematics and to Head of Department at the University of Leicester. Here, he arrived with a mandate to elevate the department’s strength in applied mathematics and computing. With the addition of brilliant young experts to the faculty, he succeeded astonishingly well in this endeavor, which was still in progress at the time of his death. In 1998, Will

became Dean of the Faculty of Science, from which position he supervised the creation of a Center for Mathematical Modeling. His enthusiasm and leadership led to his selection as a Pro-Vice Chancellor of the University for a 4-year term; it would have involved him for 3 more years had he lived. Among his responsibilities in this position was the overseeing of research throughout the university.

Will's principal mathematical interests were the subject of approximation theory. He paid particular attention to algorithms for producing approximations, especially in situations where there are many independent variables. This topic impinges on the subject of neural networks, which attempt to mimic the workings of the human brain and the process of learning. Some of his research was motivated by problems that arise in solving partial differential equations numerically.

The use of tensor products of spaces for approximation purposes leads to interesting questions of "proximality" (existence of best approximations) and minimal projections. There is also the question of establishing the efficacy of algorithms such as the Diliberto–Strauss–von Neumann algorithm and the Pólya algorithm in multivariate approximation. Will's papers in the period 1980–1990 emphasize this branch of approximation theory. His co-authors here were Manfred von Golitschek, Nira Dyn, Carlo Franchetti, Ward Cheney, John Gilbert, Julie Halton, Sue Holland, Lin Sulley, George Phillips, and John McCabe.

Will was very influential in promoting the use of radial functions for approximation, both in Euclidean spaces and on spheres. He also worked on employing "ridge functions" in concrete approximation problems. A focus of his work in the 1990s, co-authored mainly with Yuan Xu and Jeremy Levesley, was the use of convolution kernels for generating approximations. A key part of this work was to furnish constructive proofs for the density of families of ridge and radial functions. The construction of convolution kernels had applications in the analysis of quasi-interpolation procedures.

In 1992, Will and Rick Beatson showed that good approximations could be obtained using Gaussian basis functions for quasi-interpolation, even though constant functions could not be exactly reproduced. He and co-workers (1992) showed how the compactness conditions on basis functions required in the theory of de Boor and Jia could be dropped in favor of decay conditions at infinity.

Prior to 1990, quasi-interpolation in more than one variable required an infinite grid, which was unsatisfactory for approximation on a compact domain. Will and Rick Beatson in 1993 showed how to retain approximation order on the square when truncating infinite quasi-interpolatory sums by adding special edge functions.

At this stage, Will was becoming more interested in interpolation, and in the variational framework for interpolation instituted by Golomb and Weinberger. He wished to broaden the range of functions that could serve. He reached this goal with his Ph.D. student Henry Wayne (1999), and provided error estimates for a wide range of conditionally positive definite functions. The functions covered by these estimates were described in terms of properties of their Fourier transforms.

Will was interested in extending the pointwise error estimates obtained with Wayne to L_p -type estimates. This required extension theorems and descriptions of

functions without recourse to the Fourier transform. With Jeremy Levesley (1999) he was able to give direct definitions of many of the functions covered by previous work. Subsequently, with his Ph.D. student Michelle Vail (2002), he was able to give L_p -type estimates.

Will was also interested in radial interpolation on the sphere. Again, utilizing the Golomb and Weinberger theory, he, with Jeremy Levesley, David Ragozin, and Xingping Sun, established error estimates that gave unprecedented high-order convergence for local approximation on the sphere. The missing link was the boundedness of certain Lebesgue constants under scaling on the sphere. These bounds he established with Manfred von Golitschek (2001).

Being motivated by practical considerations of computation, Will was dissatisfied with the status of algorithms for radial interpolation. With Rick Beatson (1997 and 2000) he worked on the fast evaluation of radial basis functions and on the fast solution of the radial basis interpolation equations. Here, using a domain-decomposition algorithm, he and his co-workers were able to make use of earlier work concerning the alternating algorithm of von Neumann to prove convergence of the domain-decomposition algorithm. Preconditioned domain-decomposition iterative methods are at present ranked as “state-of-the-art” for fast computation of radial basis interpolants.

Will’s final work was in exploring the convergence of radial approximation when the function being approximated is conspicuously rough. With his Ph.D. student Rob Brownlee (2002) he was able to show that functions deficient in smoothness were optimally approximated using the smoother basis functions.

Will pioneered in the study of tensor product spaces and their approximation powers. In addition to papers on this subject in the years 1980–1987, he co-authored the book *Approximation Theory in Tensor Product Spaces* (1985, Springer). He also co-authored the post-graduate textbook *A Course in Approximation Theory* (1999, Brooks-Cole). He also wrote a textbook on analysis, *An Introduction to Abstract Analysis* (1990, Chapman & Hall).

Will’s non-mathematical interests were far-ranging. He qualified for and put to good use a coach-driver’s license during his years in Lancaster. He and Anita built two houses, one in Lancaster and the other in Illston-on-the-Hill. In both cases, they carried out a large portion of the work themselves, starting with the design and continuing through many phases to such details as the finish-work of the wood paneling. Their home, Gartree House, was of traditional rural design to harmonize with the village of Illston-on-the-Hill, but was of the finest materials and incorporated up-to-date innovations. He and Anita were avid skiers, and took advantage of the ski resorts in the Rocky Mountains or the Austrian Alps when they were able to do so. They loved the mountains and were fond of rock climbing. Will obtained an airplane pilot’s license and made frequent use of it in Leicester as well as when he was in Texas. He belonged to the Leicester Aero Club and often flew himself to engagements in Britain. He liked to take friends on excursions in the plane. Will learned to be a lay preacher, and was in demand as a guest speaker in a number of churches.

1. Doctoral students of Will Light

S.M. Holland, 1983
E.J. Halton, 1985
H.S.J. Wayne, 1997
M. Vail, 2002
R. Brownlee
D. Hunt

2. Recollections

[*Contributed by Charles Goldie*] Will Light was probably my most distinguished student and I am so shocked and dismayed by his most untimely death. I had general oversight over his progress through the 3 years of his undergraduate degree. Will was quite a wayward student, either inspired or bored by each course he took, with no reaction in between. I was therefore thrilled by his rapid rise in numerical analysis and by the distinction he acquired. Despite his having risen to Professor and Head of Department I found him when I visited to have retained the engaging optimistic manner that I remembered in him as an undergraduate, but overlaid with a confident sense of purpose that he had found in the meantime. When I last met him in May 2002 he had risen at the University of Leicester to be one of the immediate deputies to the Vice-Chancellor, and had an indefinable but clear air of authority that made me wonder if he'd be a Vice-Chancellor before very long. What a loss it is to learning and education that we will never know.

[*Contributed by Charles Clenshaw*] I came to know Will Light first when he arrived at the Department of Mathematics at Lancaster in 1972 to take our 1-year MA course in numerical analysis and approximation theory. During that year he took Donald Kershaw's lectures on approximation theory, and was inspired to pursue the subject henceforth. He stayed on to take his Ph.D. with me (his thesis was entitled "Projections in Numerical Analysis") and he became first a Tutorial Fellow, then a lecturer in the department, in which he stayed until 1991.

From his earliest days at Lancaster, Will was enthused by his subject and motivated by the practical applications, though he remained essentially a pure mathematician. His research topic arose from a suggestion of Leslie Fox, who remarked to me that we all know the expansion of a function in Chebyshev polynomials to be a good starting point for its polynomial approximation, under very general conditions. He then said that although we can furnish plausible explanations for such an expansion to be superior to rival candidates, a firm mathematical foundation is lacking. It seemed a fruitful field of investigation for a good research student. Will threw himself into it, and managed to extract specific mathematical problems to be attacked, from the original, somewhat nebulous, suggestion. After many challenging discussions he produced a thesis that formed a remarkably coherent presentation of a disparate set of results on projective polynomial approximations to continuous functions.

He went on to participate in many productive collaborations. In addition, Will was an effective and popular teacher, and I can testify from my time at Lancaster that he was highly valued as a member of the Department, always ready to shoulder responsibility. It was a pleasure, though not a surprise, to see how successful his Leicester career was proving to be, and his loss is a devastating blow to all who knew him.

[*Contributed by Graham Jameson*] Will's unfailing energy, enthusiasm and good humor are remembered by all his former colleagues at Lancaster, where he spent 19 years of his life.

Ahead of nearly all his colleagues, he recognized that TeX/LaTeX was becoming established as the prevailing typesetting system in mathematics, and he gave unofficial classes to drag the rest of us into the new age!

Donald Kershaw can claim to be the one who introduced Will to approximation theory. Will never forgot this debt. A typical illustration was seen on the occasion of Donald's 60th birthday. In those days, birthdays were not much celebrated in the department, but Will was not allowing this one to pass without notice: he found out about the date, made a cake and brought it to the department as a complete surprise.

He was physically fit, seemingly until the day he died. He regularly beat me at squash, and was my companion on energetic walks in the mountains of the English Lake District. His other climbs included the Matterhorn.

His strong Christian faith was central to his life. He assumed a prominent role in all the churches he attended, frequently preaching and conducting services.

[*Contributed by Peter Turner*] I first met Will when I arrived at Lancaster as a new faculty member in 1974. Will and I collaborated on many projects during our time together at Lancaster—especially in the realms of development of the numerical analysis and mathematics curricula and the beginnings of the SRC Numerical Analysis Summer Schools. Will was a wonderfully energetic and innovative colleague in all aspects of his profession—and a good friend who was generous with his time. His abilities were certainly not limited to his mathematical pursuits. He learned nearly all the practical skills needed to build his own house in Lancaster (and later applied his knowledge on his home in Leicester). He was an athlete who was always a thorough gentleman—and fierce competitor—as he defeated me regularly on the badminton court. Will always found time for life in his career and lived that life to the fullest extent. During one visit to me in the United States, he wanted to show us his skill as a pilot with a light aircraft tour of the Washington DC and Chesapeake Bay area. On that occasion, the weather defeated his plans—a very rare event indeed that any of Will's plans did not succeed! Will is a man who will be sorely missed by all those whose lives he touched.

[*Contributed by Jeremy Levesley*] Will had many qualities. He had ideas and energy, generosity and honesty, and was always looking for the humor in a situation. He was a pleasure to work with, both in research and in teaching. He was always trying things out to inspire the students and to broaden their education.

While exuding confidence, Will had a humility about him that was very engaging. I remember working with him at the blackboard and his remarking that it was as well there were no undergraduates there to see how bad we were at integration by parts.

He brought me into approximation theory, and was a constant source of guidance and interesting discussion. He always had a number of different research ideas cooking, usually on a low heat, but suddenly would burst into activity as he saw how to solve one of these problems. He would then work with immense industry and precision to reach his goal.

My main memory of him will be how he used to fizz into my office, bursting with some new scheme he had. He would be frustrated by all the possible obstacles in the way but never seemed daunted. I will miss the sense of the possible that he carried with him, and the energy with which he believed in a better future.

[*Contributed by Rick Beatson*] Will was a joy to be around in both research and social situations. On the social side Will and Anita looked after visiting mathematicians incredibly well, giving us a plentiful supply of food, exercise and above all friendship. On the research front, Will particularly enjoyed the excitement of the exploration phase of a research project, when what was true or untrue, and what tools should be used, were both unclear. However, he also had the strength of character to get through horribly intricate calculations. This was particularly evident in the writing of one of our papers, which shall not be identified, but whose working title was “the cure for insomnia.” Will was also a very accomplished lecturer. A colleague at an institution in Australia told me of his initial fright when Will, his guest, turned up to give an hour talk with no notes. To this colleague’s immense relief Will then proceeded to give a beautiful talk from memory with only the blackboard as an aid. I, and many others, will miss Will greatly.

[*Contributed by Manfred v. Golitschek*] It was in Austin, Texas, in January 1980, at the third Texas Conference on Approximation Theory, when Ward Cheney introduced me to Will. Ward and Will were already working jointly on the approximation of bivariate functions by sums of univariate ones, a topic that would fascinate all three of us for several years.

The first intense collaboration between Will and me came in September 1988. My family and I were able to spend that month with Will and Anita in Lancaster, supported by a SERC Grant. Will was accustomed to working very hard, even in the evenings and during the weekends, when he would try to write down the achievements very carefully. Nevertheless, Anita, my wife, Gabi, and I succeeded quite often to keep him away from work and start long discussions, or long excursions on bicycles in the rain (which is not considered to be rain in England). Sometimes we played badminton or billiards, his preferred sports.

Will loved to travel, and his favorite destinations were Texas and New Zealand, as far as I know; but he and Anita also spent 6 months in Würzburg from February to July 1997, supported by a grant of the DFG. He gave lectures to our German students, who appreciated his precise and entertaining way to teach, and he gave me lessons about “radial basis functions,” a topic in which he was one of the world’s leading experts.

He enjoyed learning foreign languages, and would practice them from the beginning, without fear. When Will and Anita were back in Leicester in August 1997, I wrote him an e-mail in English, not in German as he wished. He replied immediately and asked: “Manfred: Was ist das für eine Sprache? Wilhelm.”

Auf Wiedersehen, Wilhelm. Gabi and I miss you very much.

[*Contributed by Ward Cheney*] I am glad to have an opportunity of recording some reminiscences of Will Light. It is a catastrophe that this vigorous and influential man is no longer with us. Those two adjectives are not the only ones that come to mind in recalling Will's life. He was also responsible, hard-working, unflappable, devout, and solicitous, to name a few more of his many stellar qualities.

I met Will in 1976, and we discovered that we had a number of mathematical interests in common. With the help of a SERC Grant that he won, I was able to spend the summer of 1977 in Lancaster, where we worked on some problems in the subject of "projections." At that time Will was finishing (or had just finished) his Ph.D. work with Professor Charles Clenshaw, the dissertation being also in the area of projections in approximation theory. From 1977 on we collaborated more or less continually, depending on grants from various sources for the necessary travel. I and my wife, Victoria, were grateful for an opportunity to spend the Spring term of 1984 in Lancaster. Will and Anita spent similar longer periods of time in Texas whenever it was possible.

Our *modus operandi* in doing research was as follows: Whenever we could, we worked together at a blackboard, and polished such results as we thought publishable right then and there on the board. (Will usually allowed me to be the scribe who recorded the results.) If, however, we became stymied, I would go off and take care of other responsibilities, which in Texas meant lecturing to my classes, helping students during scheduled office hours, or attending departmental committee meetings. When I was finished, Will would have overcome our difficulty during my absence, and we could go on. Sometimes, the impediment was more intractable and I had to be absent for a longer period, even days, so that Will would have enough time to resolve whatever difficulty blocked our progress. It was clever of me to have attached myself to such a brilliant and hard-working partner. But imagine what he might have accomplished had I been totally absent!

Our collaboration flourished in the intervening years and resulted in numerous papers and two books. He and Anita found it possible to visit Texas (Austin or College Station) for longer periods, and I visited Lancaster and Leicester on numerous occasions to work with Will and his colleagues. Will and Anita always took me under their wing, to ensure I didn't suffer from malnutrition! I look back on those times with overwhelming feelings of nostalgia, and wish it were possible to turn back the clock to re-live them. If that were possible, we would rewrite the script, so that our story could have a happy ending. Will, my friend and colleague, you will be remembered by many with deep affection, and will be very much missed.

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